FAILURE RISK SIMULATION BY PROBABILISTIC FRACTURE MECHANICS AND QUANTITATIVE NDT

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FAILURE RISK ASSESSMENT

ONE POSSIBLE DEFINITION OF RISK:

$$RISK\left(\frac{Consequence}{UnitTime}\right) = Frequency\left(\frac{Event}{UnitTime}\right) \times Severity\left(\frac{Consequence}{Event}\right)$$

- **Risk frequency assessment**
  - **Statistical inference on past events (*a posteriori* analysis)**
  - **Probabilistic prediction (*a priori* analysis)**

- **Severity assessment**

Evaluations of economic, social, environmental, … political, ... nature

SOURCES OF RISK

MAIN: **Our inability to predict precisely what the future holds. This is mainly due to variability and uncertainty.**

- **VARIABILITY is the effect of chance and is function of system**

  Variability is objective since it resides in the nature of the involved physical mechanisms.

  - It is not reducible through either study or further measurements

  - It may be reduced by changing the system

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- **UNCERTAINTY is the assessor's lack of knowledge about:**

  - physical laws

  - parameters that characterize the physical system

  - semantics

  Uncertainty is reducible by further experiments and study.

- **The degree of certainty is our measure of how much we believe something to be true.**

In practice certainty is validated by positive (confirming) experiments.

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- **VARIABILITY and UNCERTAINTY act conjointly to erode our ability to predict the future behavior of a system.**

- **VARIABILITY and UNCERTAINTY are the components to be quantified in RISK assessment.**

- **VARIABILITY and UNCERTAINTY are quantified by methods pertaining to:**

  - **Applied statistics**
  - **Probability theory**
  - **Fuzzy logic ***
  - **Neural networks ***
  - **Elicitation ***

*** not treated in this presentation**

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PROBABILISTIC MODELS (PM) FOR RISK ASSESSMENT

Quantitative PM models are based on the theory of the underlying physical processes

Pure PMs introduce parameter description and their interaction by RV.

The main methods to construct PM are:

- **Multiple convolution integrals**
- **Markov chains**
- **Bayesian inference**
- **Monte Carlo random simulation**

Not to forget ! Physical models, PMs included, are idealizations of reality. Hence, all models are false.
However, by improvements, models can approach reality as close as possible.

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TOOLS TO QUANTIFY FAILURE RISK IN STRUCTURAL ELEMENTS

1. **Quantitative Nondestructive testing (QNDT)**
   - Flaws detection and their quantitative evaluation
   - Quantify uncertainty (or reliability) associated with a specific NDT procedure by means of probability of detection (POD)

2. **Probabilistic Fracture Mechanics (PFM)**

   One way of approach relates :
   - loading
   - material strength characteristics
   - flaws size

   as random variables (RV) in fracture mechanics rules.

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PROBABILISTIC FRACTURE MECHANICS (PFM) - Why ?

- Offers computational tools for quantifying in an "*a priori* " and/or "*a posteriori* " analysis the:
  - FAILURE RISK - in terms of failure probabilities P_f
  - RELIABILITY - in terms of survival probabilities P_s

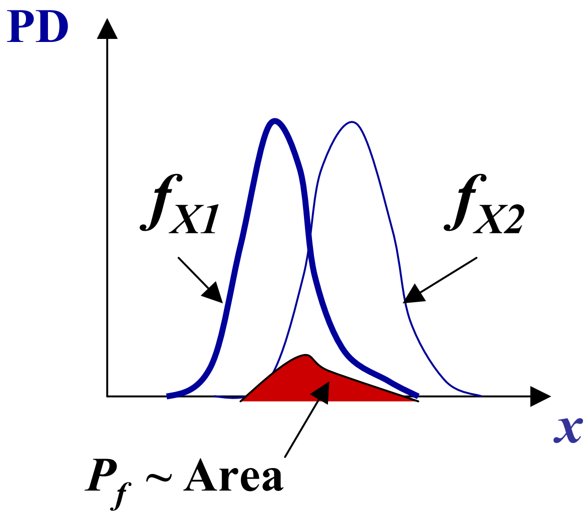
    $P_f = 1 - P_s$
- PFM (one way of approach) encompasses and accounts on basic variability and uncertainty encountered in structural design and operation:
  - Material fracture and deformation characterization: UTS, YP, Kc,
  - External loading Smax characterization
  - Defects size, a, orientation, ϕ and location, l
  - Incorporates the reliability of non-destructive testing - POD
  - Environmental induced damage
  - Human errors / reliability (tentatively)

SOME METODS OF PROBABILISTIC FRACTURE MECHANICS

1. MULTIPLE CONVOLUTIONS INTEGRAL

- Fracture model described in X_i random Variables (RV) related by model function: $Z = z(X_1, X_2, \dots X_n)$
- Failure is defined by limit state (performance) condition: $Z(.)=0$
- Probability of failure results as: $P_f = Prob\, [\, z(X_1, X_2, \dots X_n) < 0\,]$ or by the convolution integral:

$$P_f = \int \dots \int_{Z(.)<0} f_X(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n$$



$f_X = (x_1, x_2, .. x_n)$ - Joint probability density (PD) function of $X_1, X_2, \dots$

$f_X(.)$ - In practice it is difficult to be defined as a joint PD function

- A simplification assumption: *X1, X2*, ... independent RV

$$f_X = f_{X_1}(x_1) \cdot f_{X_2}(x_2) \cdot \dots \cdot f_{X_n}(x_n)$$

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PFM APPLICATION OF MULTIPLE CONVOLUTIONS INTEGRAL METHOD

- ***Z = K$_c$ - K*** **: is the model function. Fracture toughness *Kc* and SIF *K are independent RV* with *$f_c(K_c)$* and *$f_K(K)$* PD functions**
- **Probability of failure as convolution integral:**

$$P_f = \iint\limits_{K_c - K = 0} f_c(K_c) \cdot f_K(K) dK_c dK$$

- **From NDE: *$f_a(a)$* the PD function of the crack size.**
- **Common assumption: *$f_a(a)$ as* exponential distribution : with $\overline{a}$ mean crack size**

$$f_a = \left(1/\overline{a}\right)\exp\left(-1/\overline{a}\right)$$

- **PD transformation: from *f_a (a)* to *f_K (K)* :**

$$f_K(K) = f_a(a(K))\frac{da}{dK}$$

- ***f_K (K)* results as Rayleigh distribution :**

$$f_K(K) = \left(2K/\overline{K}^2\right)\exp\left(-K/\overline{K}^2\right)$$

- **POD / PND implementation (Marshall rule) :**

  Prob (a < crack size a+da) = fa(a)PND(a)

$$PND(a) = (1-A) + A\exp\left(-a/a_1\right)$$

- **Probability of failure (compact solution):**
  K_{co} = deterministic lower threshold

$$P_f = (1-A)\exp\left(-\frac{K_{co}^2}{\overline{K}^2}\right) + A\left(1+\frac{\overline{a}}{a_1}\right)\exp\left[-\left(1+\frac{\overline{a}}{a_1}\right)\frac{K_{co}^2}{\overline{K}^2}\right]$$

SIF corresponding to the mean crack size $\overline{a}$: $\overline{K} = Y\sigma\sqrt{\pi\overline{a}}$

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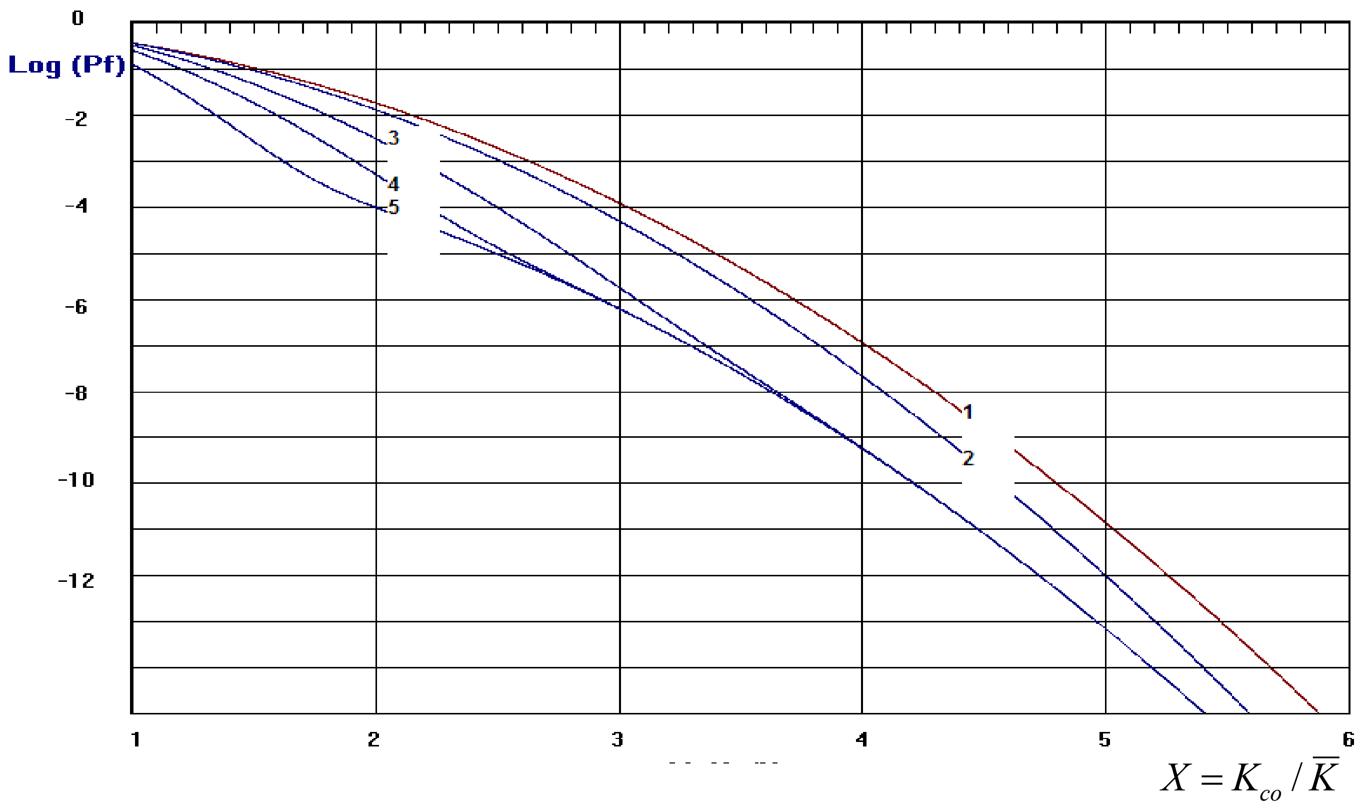


Influence of NDE on failure probability.

K_o=60MPa√m, a*=8.85mm, *A=0.995, Y=1.12.*

Numbered curves: 1- no-NDE; 2 to 5 with NDE; $\overline{a}$ = *1mm* (2); $\overline{a}$ = *5mm* (3); $\overline{a}$ = *10mm* (4); $\overline{a}$ = *20mm* (5).

2. MONTE CARLO RANDOM SIMULATION

- For each input RV a value is sampled at random from its distribution
- The randomly sampled input variables are used to calculate in one iteration a value of the dependent variable in the physical model
- By repeating many iterations, a histogram of the dependent variable ma y be constructed as an approximation of the PDF
- By increasing the number of iterations the accuracy is improved

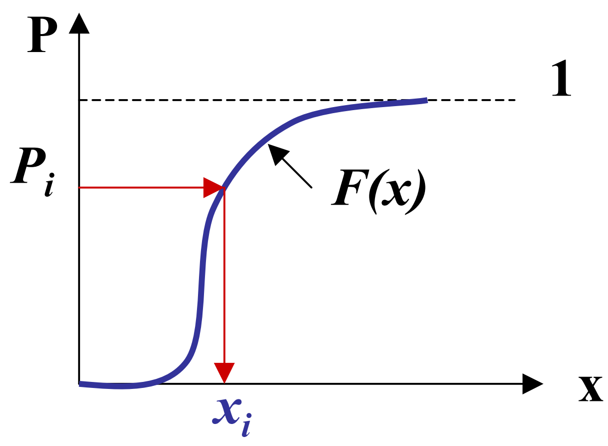
Generation of a random *x* value:

$P = F(x)$ Repartition Function (RF)

$P_i = RND$

RND uniformly distributed (0,1) RV

$x_i = F^{-1}(P_i)$: inverse of $F(x)$



Estimation of failure probability P_f

n : nb. iterations; n_f : nb. scenarios with failure

$P_f = n / n_f$

ACCURACY OF MONTE CARLO RANDOM SIMULATION

- **Coefficient of variation – COV – measure of accuracy**
- **Monte Carlo random sampling regarded as a Bernoulli trial governed by Binomial distribution**

$$COV(\overline{P}_f) \cong \frac{1}{\overline{P}_f}\sqrt{\frac{(1-\overline{P}_f)\overline{P}_f}{n}}$$

$\overline{P}_f$ **- Estimated probability of failure**

n **- Number of simulations**

Probability of failure	10^{-3}	10^{-4}	10^{-5}	10^{-6}
***n*- number of simulations at 10% COV**	10^{5}	10^{6}	10^{7}	10^{8}

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MECHANICAL DAMAGE AND FAILURE RISK ASSESSMENT

FAD: Failure Assessment Diagram
FE: Finite Elements
NDT: Non-destructive Testing
SIF: Stress Intensity Factor

Software

Expert Opinions

Geometry

- Analytical SIF
- Numeric SIF (by FE)

Reliability of NDT

- Probability of detection (POD) - models
- Probabilistic modelling human errors by NDT

Probabilistic modeling of failure risk

- Global / multicomponents analysis
- Local analysis

Failure Criteria

- Elastic-plastic FAD
- Fracture Mechanics modelling of cumulative danmage
- Phenomenologic/heuristic cumulative damage rules

Probabilistic tools

Frequentist, Bayesian, Markov chains

Flaws

- Geometry models
- Statistics of flaws size, location, orientation and density
- Statistics of flaws type

Material Strength

- Test statistics of fracture toughness Kc, UTS, and yield strength YP
- Statistical modelling of strength by “Local Approach”
- Time-dependent strength distribution shift (ageing)

Loading

- In-service loading recording
- Loading spectra processing
- Standard loding spectra

FAILURE RISK ASSESSMENT

OUTLINE OF *FRACTRISK* RATIONALE AND SOFTWARE. Principles and methods.

- **Probabilistic Fracture Mechanics** (PFM) under static and fatigue conditions
  - Elastic-plastic limit state according to *Failure Assessment Diagram* (FAD)
  - Fatigue crack growth (FCG) under short- and long-crack regime simulation. Crack closure effect
  - Equivalent initial flaw (EIF) concept for fatigue total life prediction.
- **Stress intensity factors** (SIF) defined by analytical solutions and by points (FE).
- **Cyclic loading** : constant amplitude (CA), in repeated blocks of step loading, random stationary and quasi- stationary random.
- **Integrated probability of detection** (POD) simulation. Influence of NDE reliability.
- **Monte-Carlo simulation** of input variables and automatic repetition of simulation scenarios.
- **Statistical treatment** and probabilistic distribution fitting of simulated data. Order statistics and bootstrap re-sampling.
- **Sensitivity analyses.**

FRACTRISK – PROBABILISTIC INPUT VARIABLES

□ – implemented; □ – not-implemented

Distribution	Probabilistic input				
	Material characteristics UTS, YP, Kc [1)]	FCG characteristics C, m, Ktho, B, m_1 [3)]	Crack size a, b	Loading static and cyclic	POD rule
Normal	?	?	?	?	?
Log-Normal	?	?	?	?	?
3P-Weibull	?	?	?	?	?
1/x 3P-Weibull [2)]	?	?	?	?	?
Extremal Type I	?	?	?	?	?
Extremal Type II	?	?	?	?	?
Extremal Type III	?	?	?	?	?
Logistic	?	?	?	?	?
Log-logistic	?	?	?	?	?
Rayleigh	?	?	?	?	?
Exponential	?	?	?	?	?
Uniform	?	?	?	?	?
Pareto	?	?	?	?	?

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RESULTS OF FATIGUE DAMAGE SIMULATION OBTAINED WITH *FRACTRISK*

- Fatigue life, *Nf* , for growing initial (macroscopic) crack until failure
- Fatigue life , *Ni* , at the transition from short- to long- crack FCG propagation
- Total fatigue life *Nft* = *Ni* + *Nf* by application of EIF concept. Endurance (Wöhler) curves
- Fatigue life at given crack size *N(a)*
- Remnant strength function of number of loading cycles, *R(n)*
- Fatigue life at given remnant strength *N(R)*
- Crack size failure and at given life *a(N)*
- EIF size, *ao* , by back computation from known total fatigue life *Nft*
- Fitting probabilistic simulated data to statistical distributions
- Probability of failure, *Pf* , at a given life.
- Influence of applied NDE (POD) on *Pf*
- Statistic distribution fitting to probabilistic simulated data

COLATERAL RESULTS

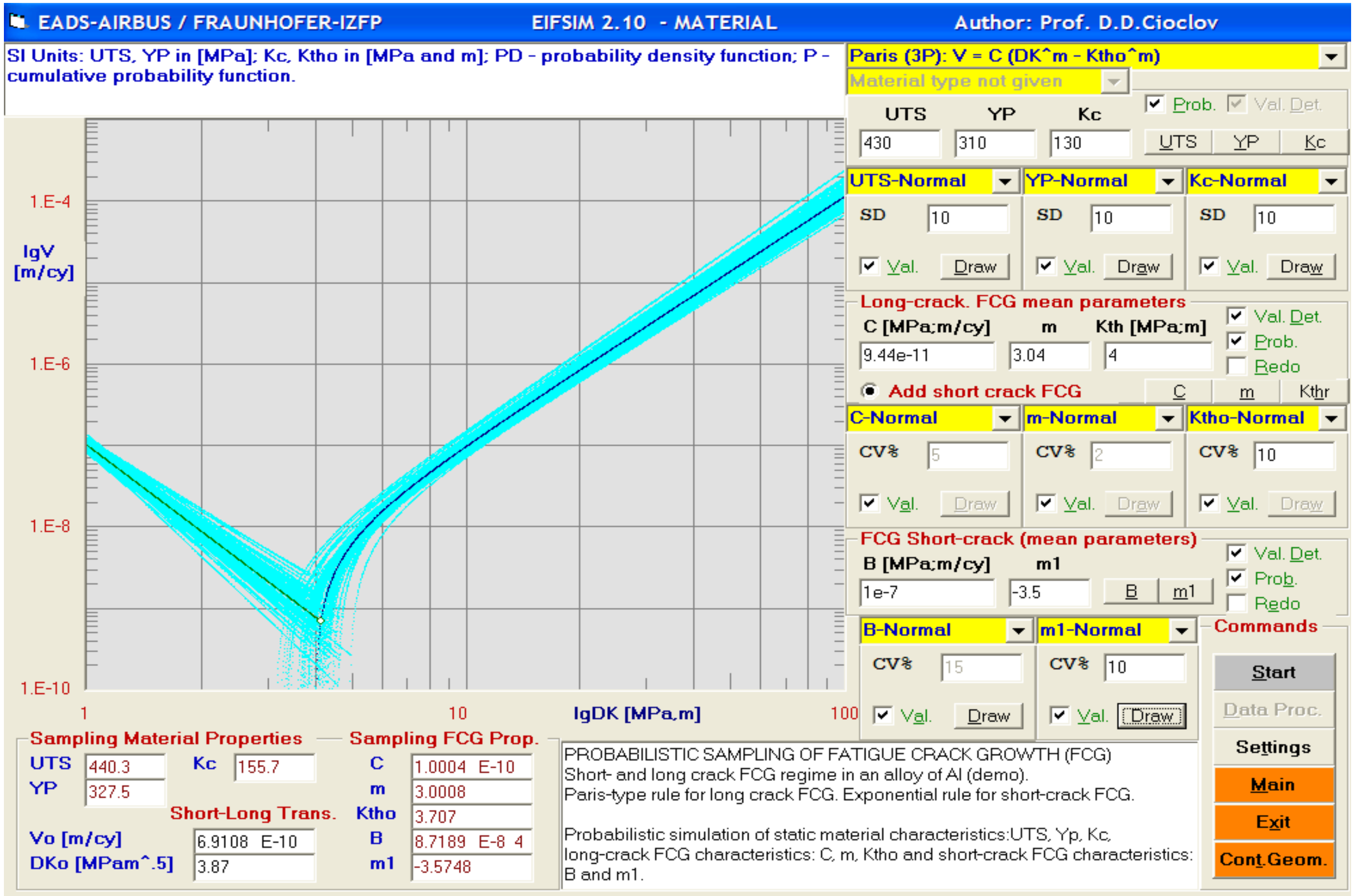
FCG curves; SIF correction factors. Random loading characteristics (e.g. root mean square). FAD deterministic sensitivity analysis. POD data fitting. Bootstraping POD data.

FRACTRISK – PROBABILISTIC OUTPUT VARIABLES

Fitting Distribution	Simulated parameter			
	N_f, N_{ft}, N_i	*N(a)*	*a(n)*	*R(n)*
Normal	?	?	?	?
Log-Normal	?	?	?	?
3P-Weibull	?	?	?	?
1/x 3P-Weibull [3)]	?	?	?	?
Extremal Type I	?	?	?	?
Extremal Type II	?	?	?	?
Extremal Type III	?	?	?	?
Logistic	?	?	?	?
Log-logistic	?	?	?	?
Exponential	?	?	?	?

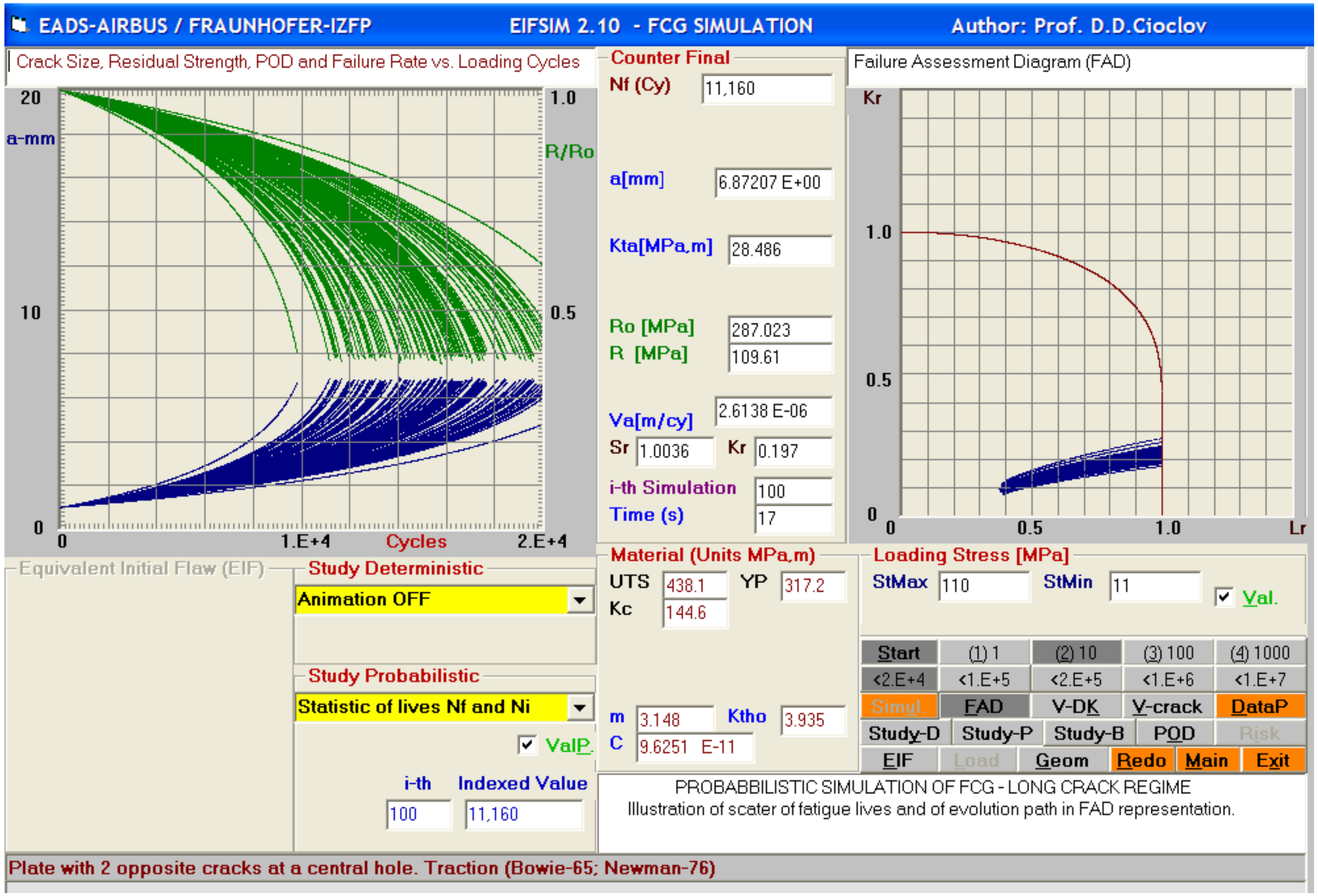
□ **– implemented;** □ **– not-implemented**

FAILURE RISK ASSESSMENT



Example of graphical display in the module for probabilistic input of material characteristics (conventional static and for FCG). Data are representative for an aluminum alloy of 2024-T3 class.

FAILURE RISK ASSESSMENT



Probabilistic fatigue crack growth (FCG) simulation until failure. Long-crack regime. Paris rule. Representation *a(n)* - crack size and *R(n)/Ro* vs. number of applied loading cycles *n* (left) and FCG path in FAD diagram.

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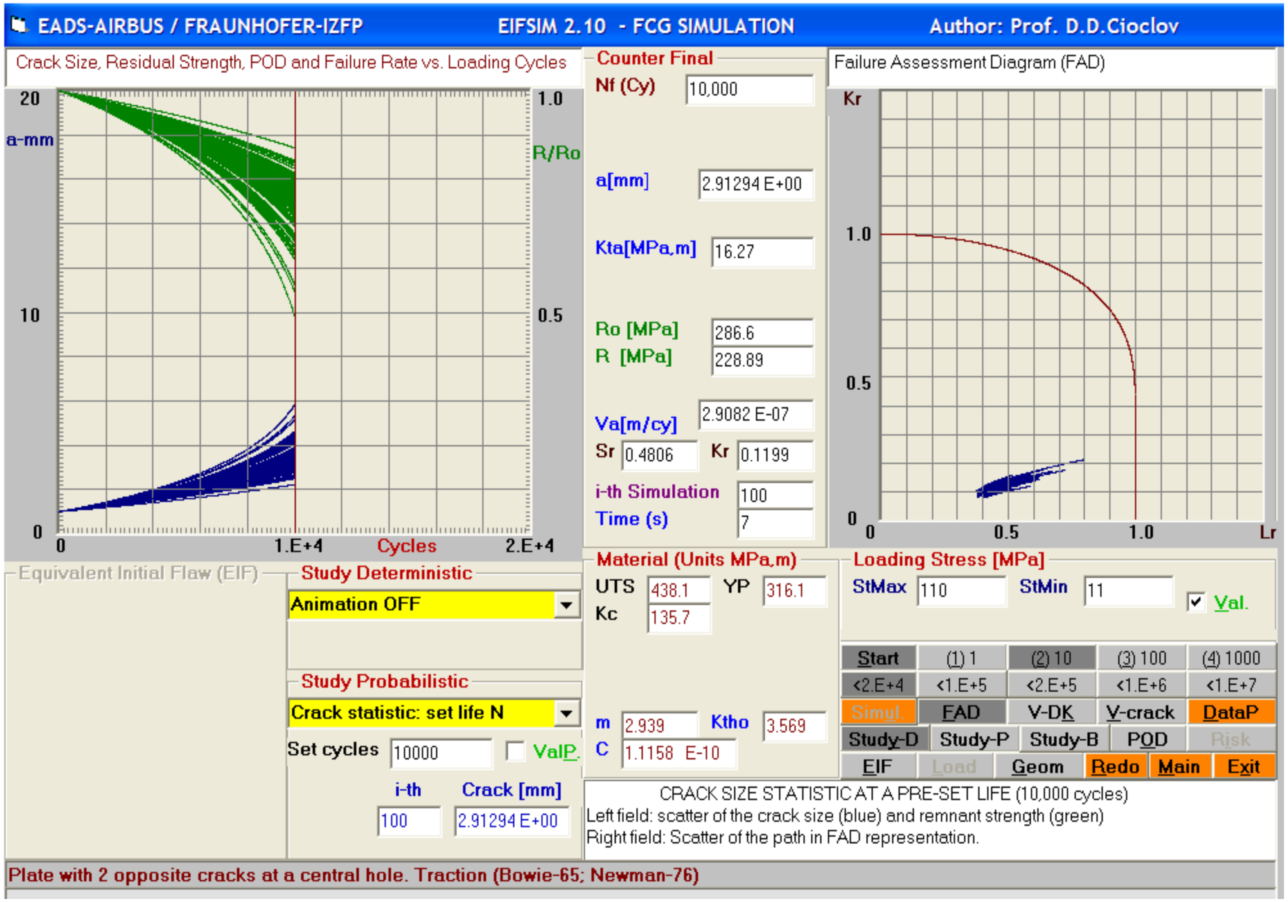

Probabilistic fatigue crack growth simulation until a pre-set number of loading cycles (10,000 cycles).

Long-crack regime. Paris rule.

Statistics of the crack size *a(n)* and remnant strength *R(n)/Ro* at a pre-set number of loading cycles.

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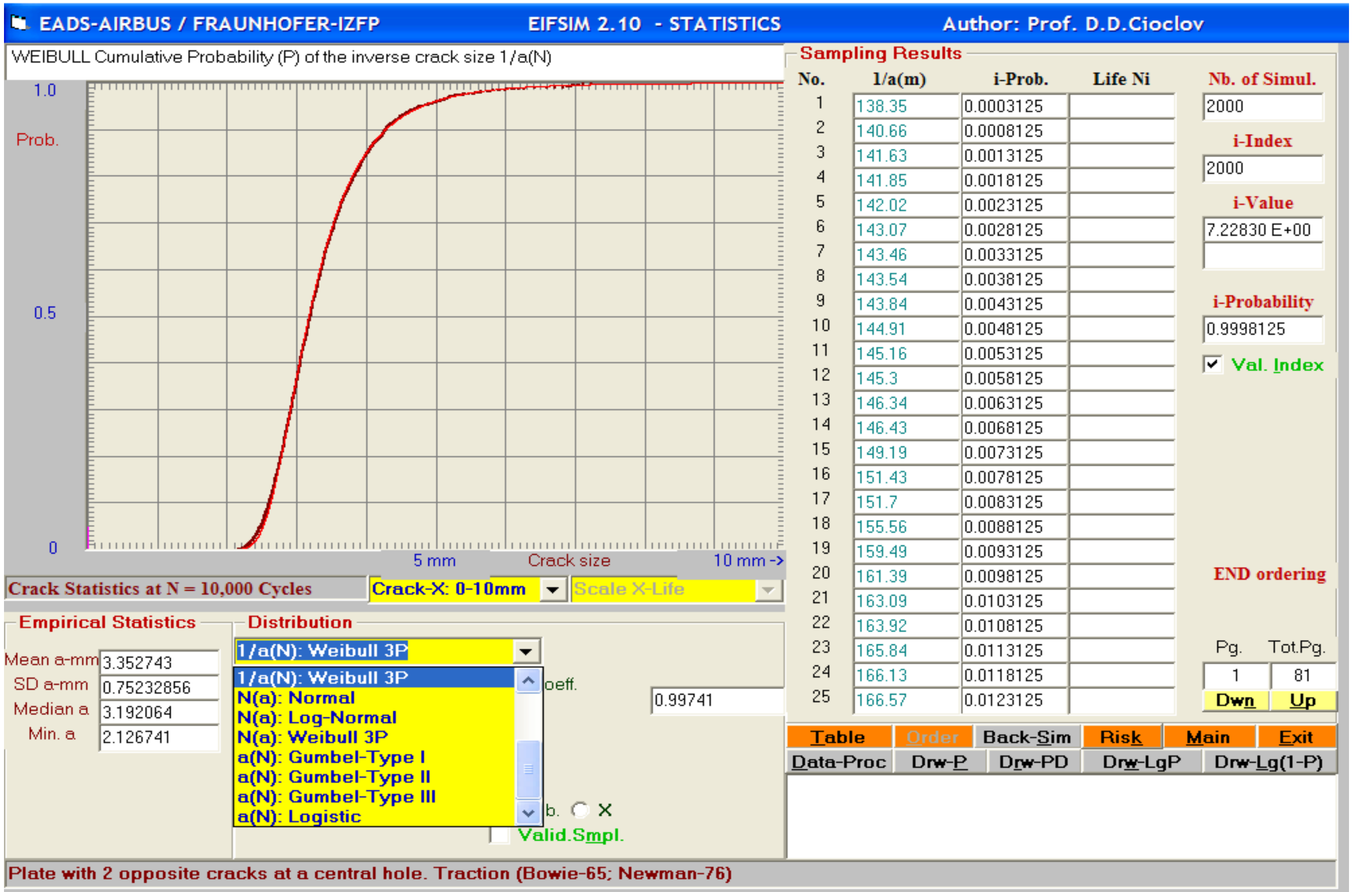

Simulated crack size data (2000 items) fitting according to 2P Weibull distribution applied to the inverse value (*1/a*) of the crack size. Cumulative probability (*Prob)* representation vs. crack size (*a*).

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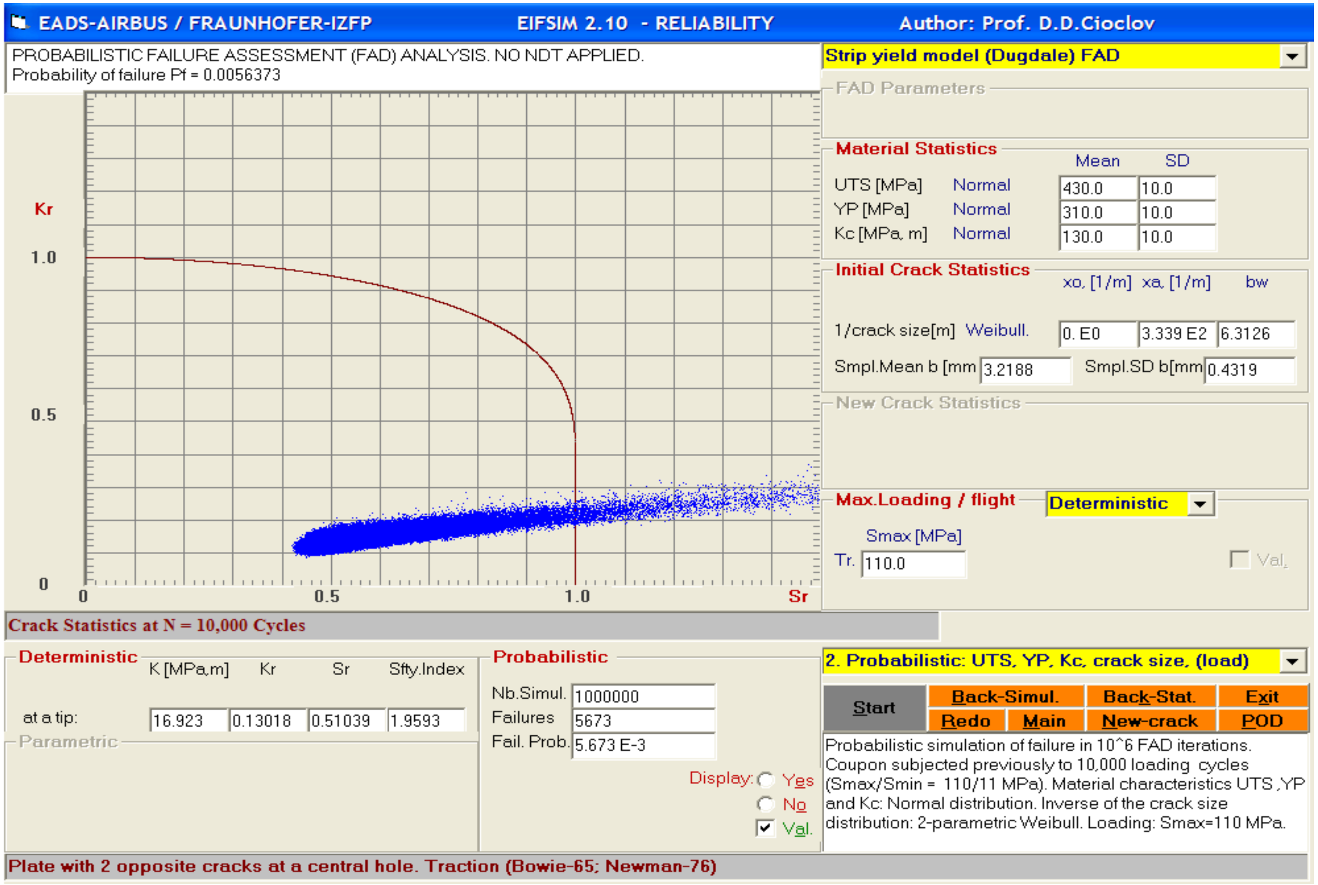

Probabilistic FAD analysis. Fatigue crack population resulted after 10^4 loading cycles. Crack size distribution: 2P Weibull of (1/a). Strength characteristics distribution: Normal. Probability of failure estimation in the next loading cycle. 10^6 Monte Carlo repetitions. The case when no NDE is applied.

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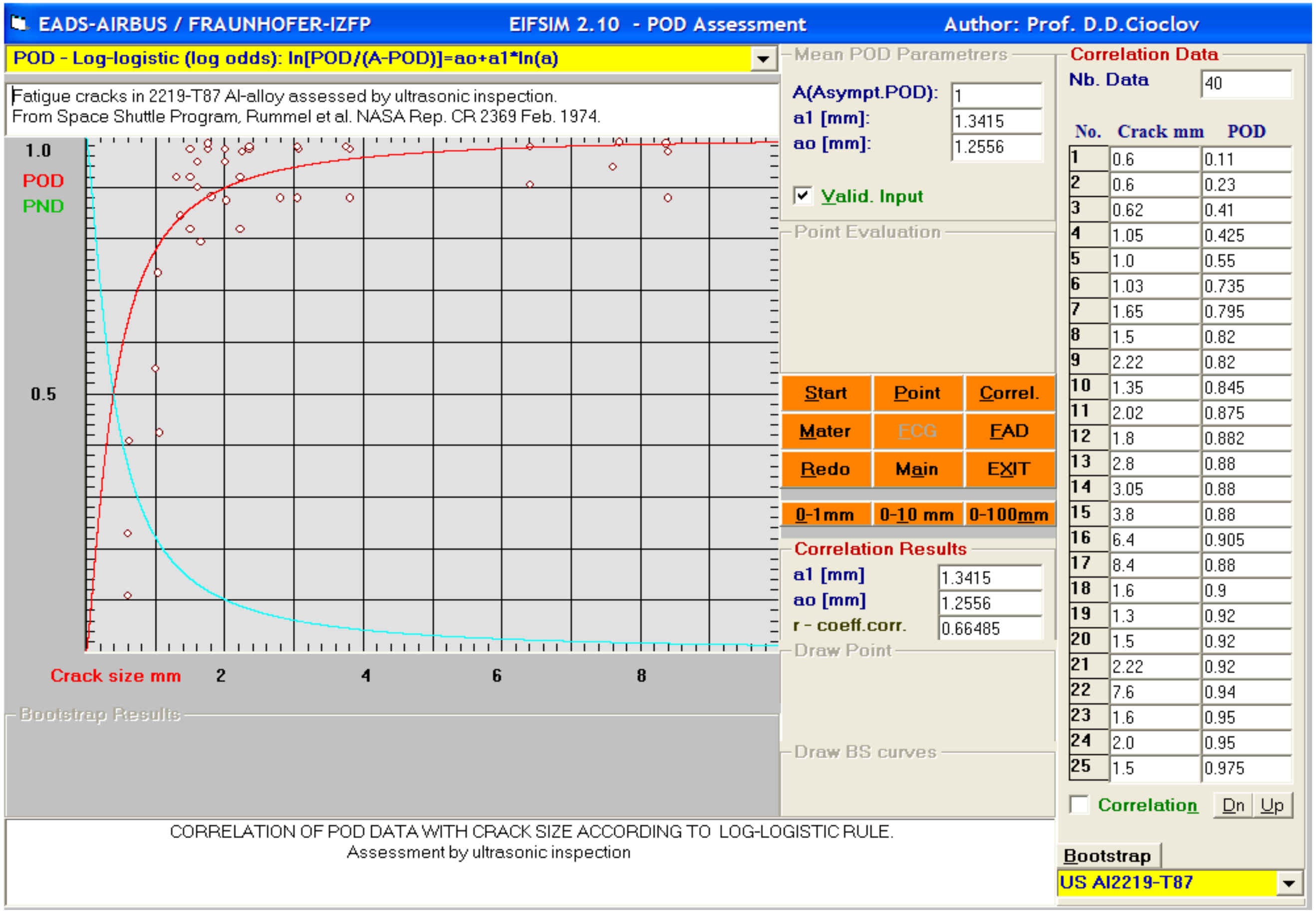

POD data correlation with Log-Logistic POD rule. Al-alloy of type T2219-T87. Ultrasonic NDE (Rummel et al. 1974).

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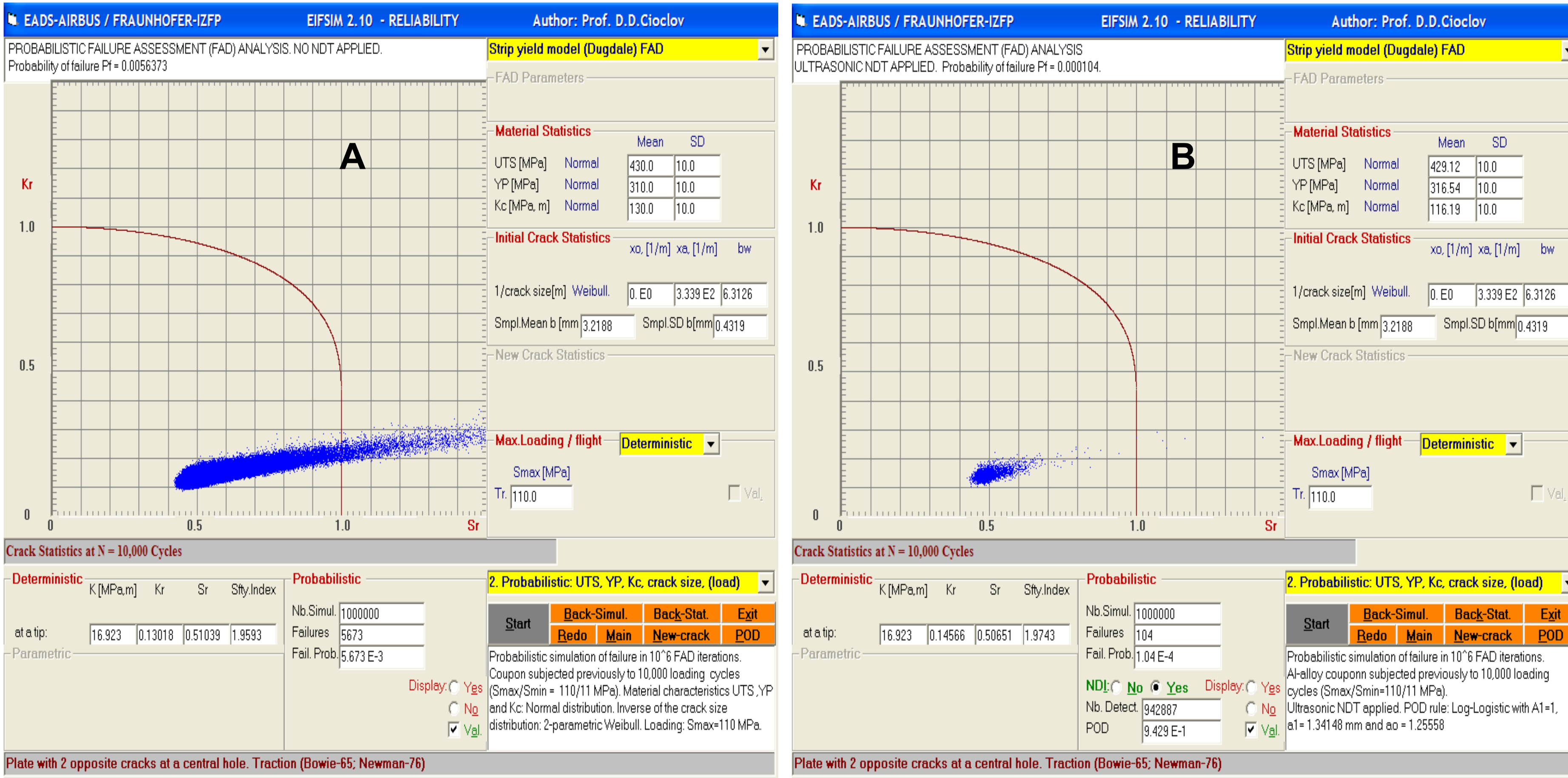

Probabilistic FAD analysis (10^6 iterations). Fatigue crack population after 10^4 loading cycles. Influence of applied NDE. Probability if failure in the next loading cycle: A – NDE not applied $P_f = 5.673\ 10^{-3}$. B – US NDE applied $Pf = 1.04\ 10^{-4}$.

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CONCLUSIONS

- The study develops a methodology for integrating probabilistic fracture mechanics with quantitative NDE for the purpose of failure risk assessment in load-carrying structures and machine elements.
- Uncertainty and variability are the main sources of failure risk
- Probabilistic methods quantifies U&V
- Two probabilistic models for failure risk assessment are developed. One, in terms of convolution integral computation, the other on the base of Monte Carlo simulation. Merits and shortcomings are outlined
- Both models enable to account on probability of detection that is specific for the applied NDE
- A rationale and a software is presented for the purpose of probabilistic fatigue damage simulation. Fatigue crack growth under short- and long- crack regime is implemented.
- Probabilistic simulated data are fitted into statistical distributions
- Probabilistic FAD analysis enables to estimate the failure probability at a specific fatigue life. It is quantified the benefit by reducing probability of failure by applying NDE (POD)
- An example is outlined for a simple configuration of interest for aeronautical structures

Computer simulation of failure risk enables a deeper insight on structural reliability, assists decisions concerning the inspection quality and timing and leads to cost reductions in engineering analysis.